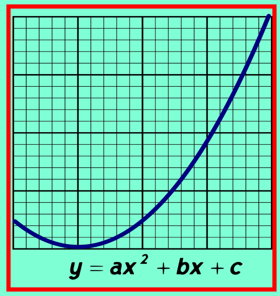
Math 125

Fall 2021

Lecture 29



Class QZ 23

Given

$$\begin{cases} 3x - 2y + z = 16 \\ 2x + 3y - z = -9 \\ x + 4y + 3z = 2 \end{cases}$$

Find D, the determinant of coef. matrix.

Always

$$D = \begin{vmatrix} 3 & -2 & 1 \\ 2 & 3 & -1 \\ 1 & 4 & 3 \end{vmatrix} = 3\begin{vmatrix} 3 & -1 \\ 4 & 3 \end{vmatrix} - (-2)\begin{vmatrix} 2 & -1 \\ 1 & 3 \end{vmatrix} + 1\begin{vmatrix} 2 & 3 \\ 1 & 4 \end{vmatrix}$$

$$= 3(9+4) + 2(6+1) + 1(8-3)$$

$$= 3\cdot 13 + 2\cdot 7 + 1\cdot 5 = \boxed{58}$$

Solve by matrix method:

$$\begin{cases} 4x - 3y = -15 \\ x + 2y = -1 \end{cases} \Rightarrow$$

Augmented Matrix

$$\left[\begin{array}{cc|c} 4 & -3 & -15 \\ 1 & 2 & -1 \end{array}\right] \rightarrow \left[\begin{array}{cc|c} 1 & 0 & \\ 0 & 1 & \end{array}\right]$$

$R1 \leftrightarrow R2$

$$\left[\begin{array}{cc|c} 1 & 2 & -1 \\ 4 & -3 & -15 \end{array}\right]$$

$(-4)R1 + R2 \rightarrow R2$

$$\left[\begin{array}{cc|c} 1 & 2 & -1 \\ 0 & -11 & -11 \end{array}\right]$$

$R2 \div (-11) \rightarrow R2$

$$\left[\begin{array}{cc|c} 1 & 2 & -1 \\ 0 & 1 & 1 \end{array}\right]$$

$(-2)R2 + R1 \rightarrow R1$

$$\left[\begin{array}{cc|c} 1 & 0 & -3 \\ 0 & 1 & 1 \end{array}\right] \quad \begin{array}{l} x = -3 \\ y = 1 \end{array}$$

$\Rightarrow$ Final Ans $(-3, 1)$

Solve by matrix method:

$$\begin{cases} 3x + y + 2z = 31 \\ x + y + 2z = 19 \\ x + 3y + 2z = 25 \end{cases}$$

Augmented Matrix

$$\left[\begin{array}{ccc|c} 3 & 1 & 2 & 31 \\ 1 & 1 & 2 & 19 \\ 1 & 3 & 2 & 25 \end{array}\right] \rightarrow \left[\begin{array}{ccc|c} 1 & 0 & & \\ & 1 & & \\ 0 & & 1 & \end{array}\right]$$

$R1 \leftrightarrow R2$

$$\left[\begin{array}{ccc|c} 1 & 1 & 2 & 19 \\ 3 & 1 & 2 & 31 \\ 1 & 3 & 2 & 25 \end{array}\right]$$

$(-3)R1 + R2 \rightarrow R2$

$(-1)R1 + R3 \rightarrow R3$

$$\left[\begin{array}{ccc|c} 1 & 1 & 2 & 19 \\ 0 & -2 & -4 & -26 \\ 0 & 2 & 0 & 6 \end{array}\right]$$

$R3 \div 2 \rightarrow R3$

$R2 \div (-2) \rightarrow R2$

$$\left[\begin{array}{ccc|c} 1 & 1 & 2 & 19 \\ 0 & 1 & 2 & 13 \\ 0 & 1 & 0 & 3 \end{array}\right]$$

$R2 \leftrightarrow R3$

$$\left[\begin{array}{ccc|c} 1 & 1 & 2 & 19 \\ 0 & 1 & 0 & 3 \\ 0 & 1 & 2 & 13 \end{array}\right]$$

$(-)R2 + R3 \rightarrow R3$

$$\left[\begin{array}{ccc|c} 1 & 1 & 2 & 19 \\ 0 & 1 & 0 & 3 \\ 0 & 0 & 2 & 10 \end{array}\right]$$

$R3 \div 2 \rightarrow R3$

$$\left[\begin{array}{ccc|c} 1 & 1 & 2 & 19 \\ 0 & 1 & 0 & 3 \\ 0 & 0 & 1 & 5 \end{array}\right]$$

$$\left[\begin{array}{ccc|c} 1 & 1 & 2 & 9 \\ 0 & 1 & 0 & 3 \\ 0 & 0 & 1 & 5 \end{array}\right]$$

$(-2)R3 + R1 \to R1$

$$\left[\begin{array}{ccc|c} 1 & 1 & 0 & 9 \\ 0 & 1 & 0 & 3 \\ 0 & 0 & 1 & 5 \end{array}\right]$$

$(-1)R2 + R1 \to R1$

$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & 6 \\ 0 & 1 & 0 & 3 \\ 0 & 0 & 1 & 5 \end{array}\right] \quad \begin{array}{l} x = 6 \\ y = 3 \\ z = 5 \end{array}$$

Final Ans $\boxed{(x, y, z) = (6, 3, 5)}$ $\quad \{(6, 3, 5)\}$

Solve by matrix method:

$$\begin{cases} 2x + y + 2z = 18 \\ x - y + 2z = 9 \\ x + 2y - z = 6 \end{cases}$$

Augmented Matrix

$$\left[\begin{array}{ccc|c} 2 & 1 & 2 & 18 \\ 1 & -1 & 2 & 9 \\ 1 & 2 & -1 & 6 \end{array}\right]$$

$R1 \leftrightarrows R2$

$$\left[\begin{array}{ccc|c} 1 & -1 & 2 & 9 \\ 2 & 1 & 2 & 18 \\ 1 & 2 & -1 & 6 \end{array}\right]$$

$(-2)R1 + R2 \to R2$

$(-1)R1 + R3 \to R3$

$$\left[\begin{array}{ccc|c} 1 & -1 & 2 & 9 \\ 0 & 3 & -2 & 0 \\ 0 & 3 & -3 & -3 \end{array}\right]$$

$R3 \div 3 \to R3$

$$\left[\begin{array}{ccc|c} 1 & -1 & 2 & 9 \\ 0 & 3 & -2 & 0 \\ 0 & 1 & -1 & -1 \end{array}\right]$$

$R2 \leftrightarrows R3$

$$\left[\begin{array}{ccc|c} 1 & -1 & 2 & 9 \\ 0 & 1 & -1 & -1 \\ 0 & 3 & -2 & 0 \end{array}\right]$$

$$\left[\begin{array}{ccc|c} 1 & -1 & 2 & 9 \\ 0 & 1 & -1 & -1 \\ 0 & 3 & -2 & 0 \end{array}\right]$$

$(-3)R2 + R3 \to R3$

$$\left[\begin{array}{ccc|c} 1 & -1 & 2 & 9 \\ 0 & 1 & -1 & -1 \\ 0 & 0 & 1 & 3 \end{array}\right]$$

$R3 + R2 \to R2$

$(-2)R3 + R1 \to R1$

$$\left[\begin{array}{ccc|c} 1 & -1 & 0 & 3 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & 3 \end{array}\right]$$

$R2 + R1 \to R1$

$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & 5 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & 3 \end{array}\right]$$

$x=5$
$y=2$
$z=3$

$(5, 2, 3)$

Graph of $y = ax^2 + bx + c$ contains $(1,40)$, $(2,48)$, and $(3,24)$

Use matrix method to find a, b, and c.

$(1,40) \to \begin{matrix} x=1 \\ y=40 \end{matrix} \to a(1)^2 + b(1) + c = 40$

$\boxed{a + b + c = 40}$

$(2,48) \to \begin{matrix} x=2 \\ y=48 \end{matrix} \to a(2)^2 + b(2) + c = 48$

$\boxed{4a + 2b + c = 48}$

$(3,24) \to \begin{matrix} x=3 \\ y=24 \end{matrix} \to a(3)^2 + b(3) + c = 24$

$\boxed{9a + 3b + c = 24}$

$$\begin{cases} a + b + c = 40 \\ 4a + 2b + c = 48 \\ 9a + 3b + c = 24 \end{cases}$$

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 40 \\ 4 & 2 & 1 & 48 \\ 9 & 3 & 1 & 24 \end{array}\right]$$

October 21, 2021

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 40 \\ 4 & 2 & 1 & 48 \\ 9 & 3 & 1 & 24 \end{array}\right]$$

$(-4)R1 + R2 \to R2$

$(-9)R1 + R3 \to R3$

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 40 \\ 0 & -2 & -3 & -112 \\ 0 & -6 & -8 & -336 \end{array}\right]$$

$(-3)R2 + R3 \to R3$

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 40 \\ 0 & -2 & -3 & -112 \\ 0 & 0 & 1 & 0 \end{array}\right]$$

$(3)R3 + R2 \to R2$

$(-1)R3 + R1 \to R1$

$$\left[\begin{array}{ccc|c} 1 & 1 & 0 & 40 \\ 0 & -2 & 0 & -112 \\ 0 & 0 & 1 & 0 \end{array}\right]$$

$R2 \div (-2) \to R2$

$$\left[\begin{array}{ccc|c} 1 & \boxed{1} & 0 & 40 \\ 0 & 1 & 0 & 56 \\ 0 & 0 & 1 & 0 \end{array}\right]$$

$(-1)R2 + R1 \to R1$

$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & -16 \\ 0 & 1 & 0 & 56 \\ 0 & 0 & 1 & 0 \end{array}\right] \begin{array}{l} a=-16 \\ b=56 \\ c=0 \end{array}$$

$y = ax^2 + bx + c$

$y = -16x^2 + 56x$

Class QZ 24

Solve by matrix Method:

$$\begin{cases} 3x - 5y = 7 \\ x - y = 1 \end{cases}$$

Augmented Matrix

$$\left[\begin{array}{cc|c} 3 & -5 & 7 \\ 1 & -1 & 1 \end{array}\right]$$

$R1 \leftrightarrow R2$

$$\left[\begin{array}{cc|c} 1 & -1 & 1 \\ 3 & -5 & 7 \end{array}\right]$$

$(-3)R1 + R2 \to R2$

$$\left[\begin{array}{cc|c} 1 & -1 & 1 \\ 0 & -2 & 4 \end{array}\right]$$

$R2 \div (-2) \to R2$

$$\left[\begin{array}{cc|c} 1 & -1 & 1 \\ 0 & 1 & -2 \end{array}\right]$$

$R2 + R1 \to R1$

$$\left[\begin{array}{cc|c} 1 & 0 & -1 \\ 0 & 1 & -2 \end{array}\right] \begin{array}{l} x=-1 \\ y=-2 \end{array} \Rightarrow \boxed{(-1,-2)}$$